

# Holes bound to acceptors as qubits: Tunability, coherence and entanglement

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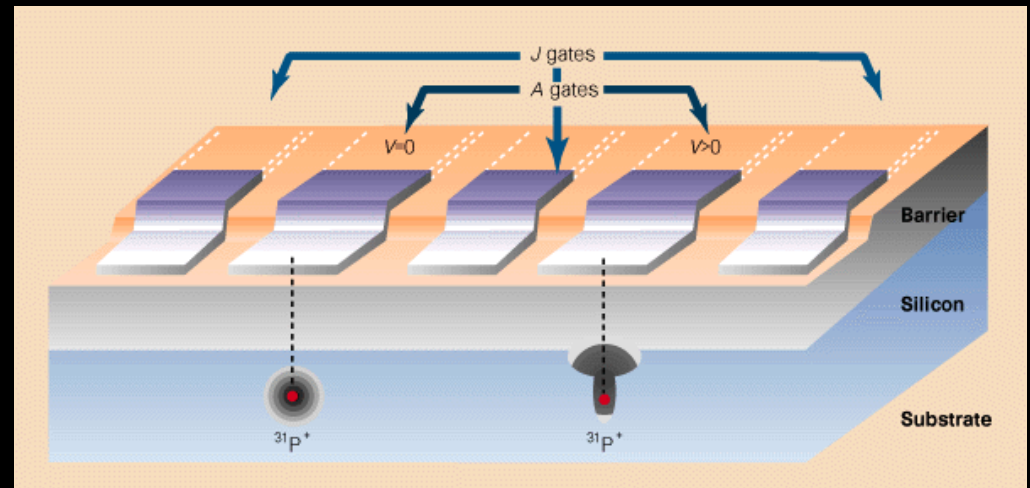
# OUTLINE

- 1. BACKGROUND & MOTIVATION
- 2. ACCEPTOR DESCRIPTION
- 3. LH ACCEPTOR QUBIT 101
- 4. IN-PLANE B FIELD. NEW PHENOMENA
- 5. EXPERIMENTAL APPLICATIONS
- 6. CONCLUSIONS

# DOPANT BASED QUANTUM COMPUTING

Proposed by Bruce Kane, Nature 393, 133 (1998).

- P donors in Si: nuclear spin  $\frac{1}{2}$
- Electron spin coupled to nuclear spin by hyperfine interaction



- A gate controls the hyperfine interaction (for 1 qubit gates)
- J gate controls the overlap of electrons (for 2 qubit gates)
- Readout: spin-to-charge conversion

# DONORS: CURRENT STATUS

- High fidelity single shot readout of electron and nuclear spin. Pla et al., Nature 489, 541 (2012). J. Pla et al., Nature 496, 334 (2013)
- Electron spin relaxation and coherence times  $> T_1$  in the order of hundreds of seconds.  $T_2 = 10$ s. Nature Materials 11, 143-147 (2012)
- Nuclear spin coherence times  $> 30$  s. Nature Nanotechnology 9, 986 (2014)
- A-gate implementation. Laucht et al. Science Advances 1, e1500022 (2015)
- Fidelities above 99.95-99.99% for single qubit operations (electron and nuclear spin respectively). J. Phys: Condens. Matter 27, 154205 (2015)

# DONORS: CHALLENGES

- Valley degeneracy: Exchange oscillations
- Small SO interaction: Slow interactions
- Oscillating Magnetic fields: Experimentally challenging and require too much power

# MOTIVATION

- Electrical spin manipulation: **spin-orbit qubits**
  - Spin-orbit enhances the coupling to electric fields
  - Single-qubit operations: EDSR
  - Also scalability e.g. cQED, dipole-dipole coupling
  - Electric fields are easier to apply and localize than magnetic fields
- Current problems
  - Spin-orbit also enhances coupling to stray fields ~ noise and phonons
  - Scalability: exchange gates vulnerable to electrical noise

# WHY ACCEPTORS?

Confinement potential is free and reproducible cf. donor

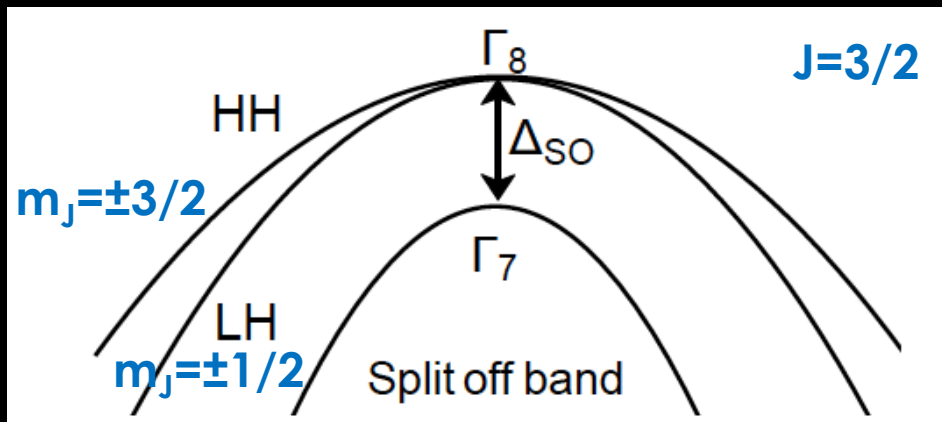
Holes have interesting properties

- Strong spin-orbit coupling in the valence band ( $L=1$ )
- Limited coupling to nuclear spins
- Effective spin-3/2 – completely different from electrons
- No valleys so no extra Hilbert space complications
- Enhanced dipole-dipole interaction
- Flexibility – can work in HH or LH manifolds

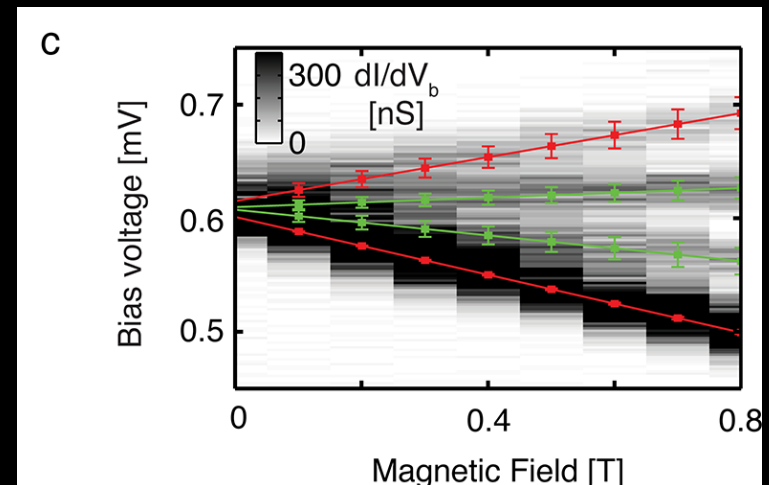


# VALENCE BANDS

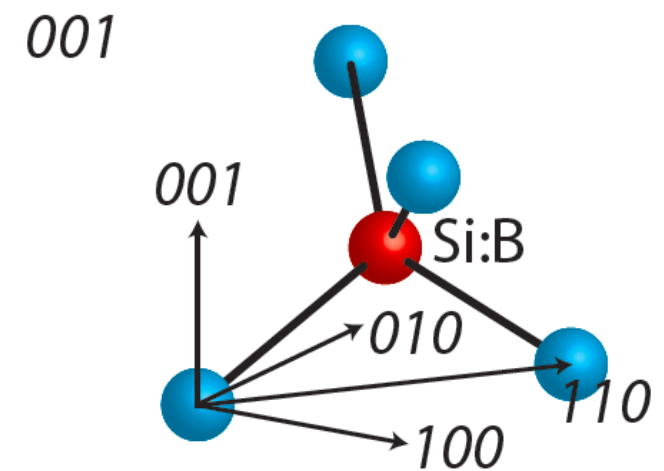
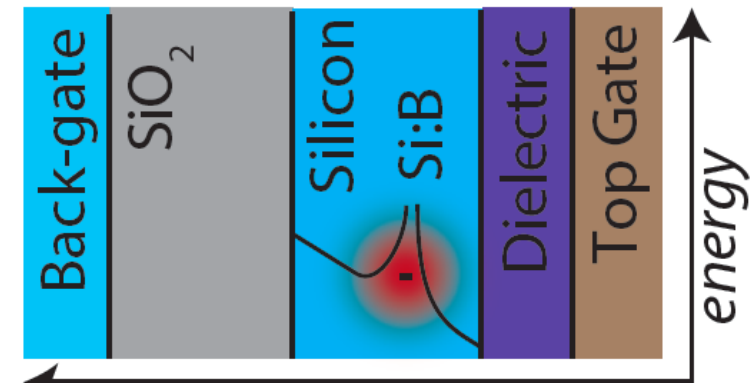
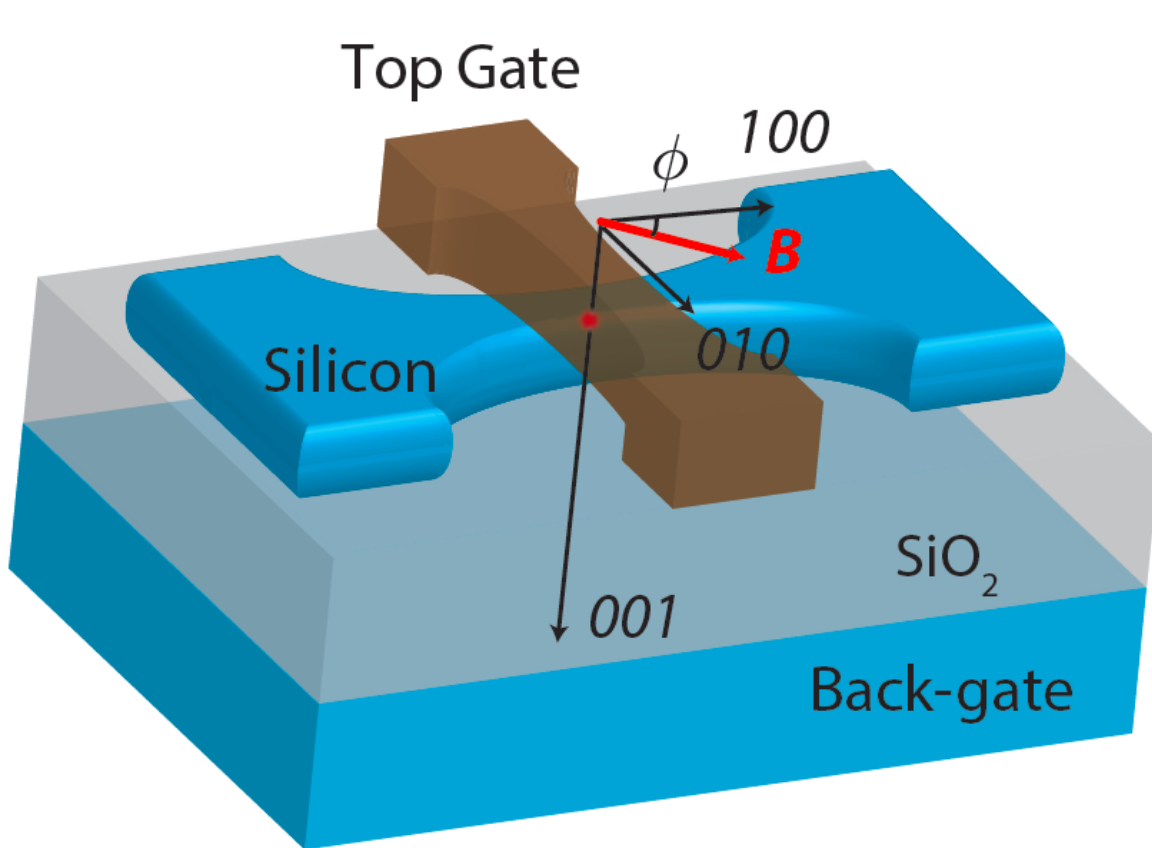
Strong spin-orbit interaction  
Effective  $J=3/2$  GS



Luttinger and Kohn, Phys. Rev. 97, 869 (1955)  
Si:B GS is 45 meV on top of VB  
1<sup>st</sup> excited state is 21 meV below



Van de Heidjen et al, Nano Letters 14, 1492 (2014)



Acceptor in Si close to a SiO<sub>2</sub> interface.

# ACCEPTOR HAMILTONIAN

$$H = H_{KL} + H_{BP} + H_c + H_{\text{inter}} + H_F + H_B + H_{T_d}$$

-Kohn-Luttinger Hamiltonian for the VB

-Bir-Pikus Hamiltonian for the strain

-Coulomb impurity

-Si/SiO<sub>2</sub> Interface

-Electric and magnetic fields

-T<sub>d</sub> symmetry of the ion

EMA+SW transformation

4x4 Effective low energy

Hamiltonian

For the 4-fold degenerate

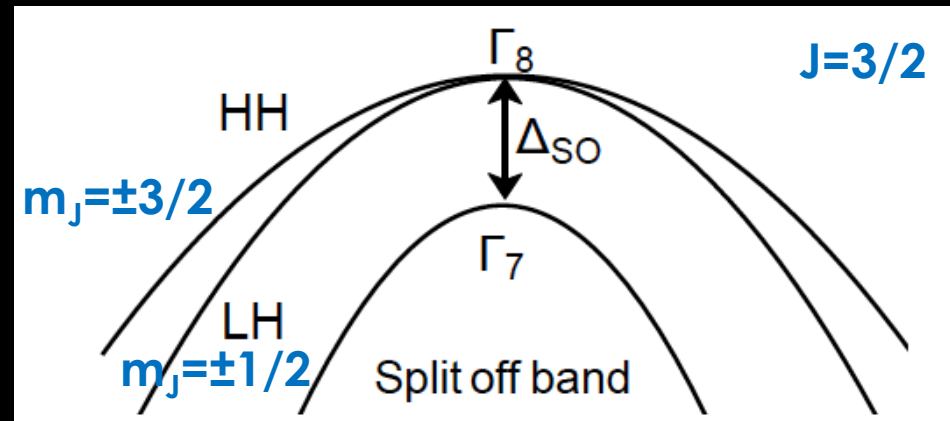
GS



# INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN:

$\{3/2, 1/2, -1/2, -3/2\}$

$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



States with  $|m_j| = 3/2$  are predominantly HH like

$|m_j| = 1/2$  are

predominantly LH like

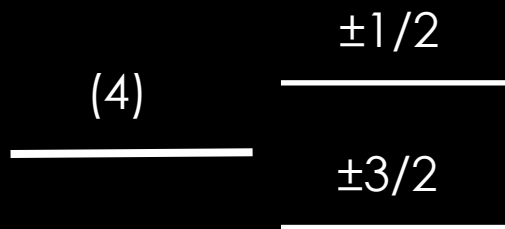
# INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN: BREAKING DEGENERACIES

$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{HL} & 0 & 0 \\ 0 & 0 & \Delta_{HL} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

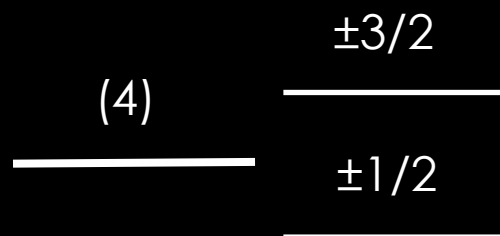
Bir-Pikus Hamiltonian

Uniaxial (001) strain

In-plane compressive  
HH ground state



In-plane tensile  
LH ground state



# INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN: BREAKING DEGENERACIES

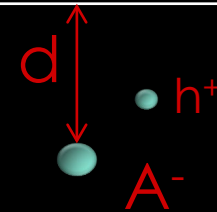
$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{HL} & 0 & 0 \\ 0 & 0 & \Delta_{HL} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum confinement +  
dielectric mismatch

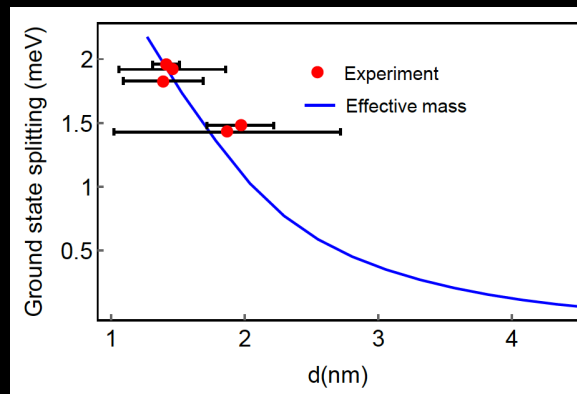
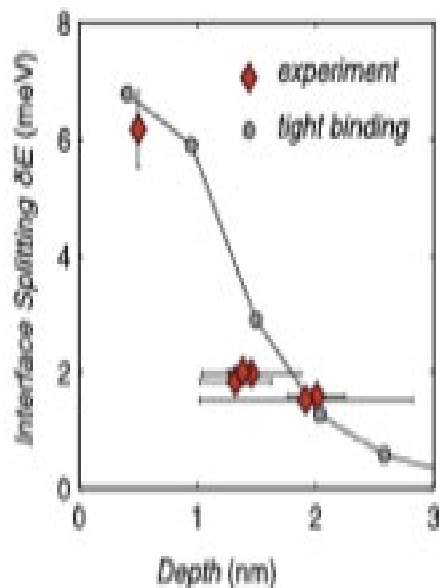
● A<sup>-</sup> image  
● h<sup>+</sup> image

SiO<sub>2</sub>

Si(001)



HH ground state



J.C. Abadillo-Uriel, M.J.C. Nanotech, 27, 024003 (2016); Mol et al APL 106, 203110 (2015)

# BUILDING THE EFFECTIVE GS HAMILTONIAN: MIXING LH-HH

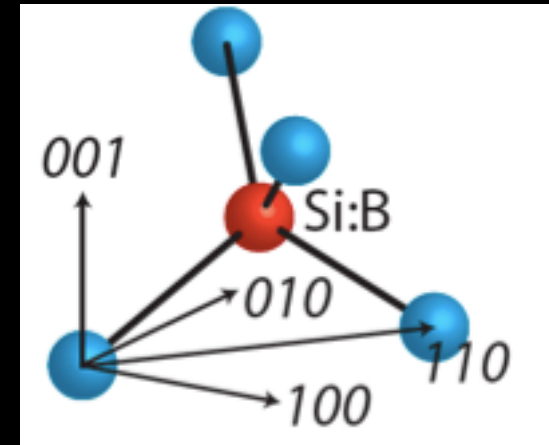
$$H_{\text{eff}} = \begin{pmatrix} \{3/2, 1/2, -1/2, -3/2\} & & & \\ 0 & 0 & -ipF_z & 0 \\ 0 & \Delta_{HL} & 0 & -ipF_z \\ ipF_z & 0 & \Delta_{HL} & 0 \\ 0 & ipF_z & 0 & 0 \end{pmatrix}$$

$$p = e \int_0^a f^*(r) r f(r)$$

This term mixes LH and HH.

p depends

- acceptor "depth"
- electric field
- distance to interface



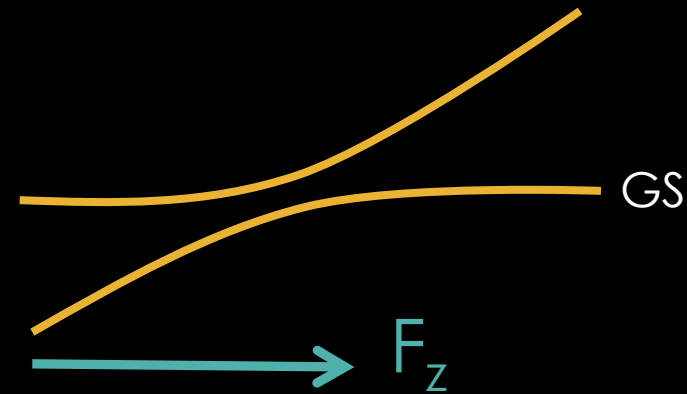
Bir, Butekov, Pikus J. Phys. Chem. Solids 24,  
1467 (1963); 24, 1475 (1963)

# BUILDING THE EFFECTIVE GS HAMILTONIAN: BROKEN DEGENERACY + MIXING LH-HH

2 branches (Kramers doublets):

$$E_u = \frac{1}{2}(\Delta_{HL} + \sqrt{\Delta_{HL}^2 + 4p^2 F_z^2})$$

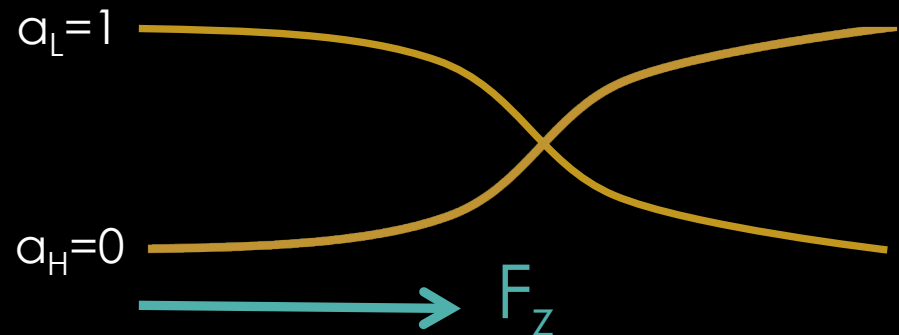
$$E_l = \frac{1}{2}(\Delta_{HL} - \sqrt{\Delta_{HL}^2 + 4p^2 F_z^2})$$



The qubit is defined in the lower branch with probability amplitudes on LH and HH:

$$a_L = E_l / \sqrt{E_l^2 + p^2 F_z^2}$$

$$a_H = pF_z / \sqrt{E_l^2 + p^2 F_z^2}$$

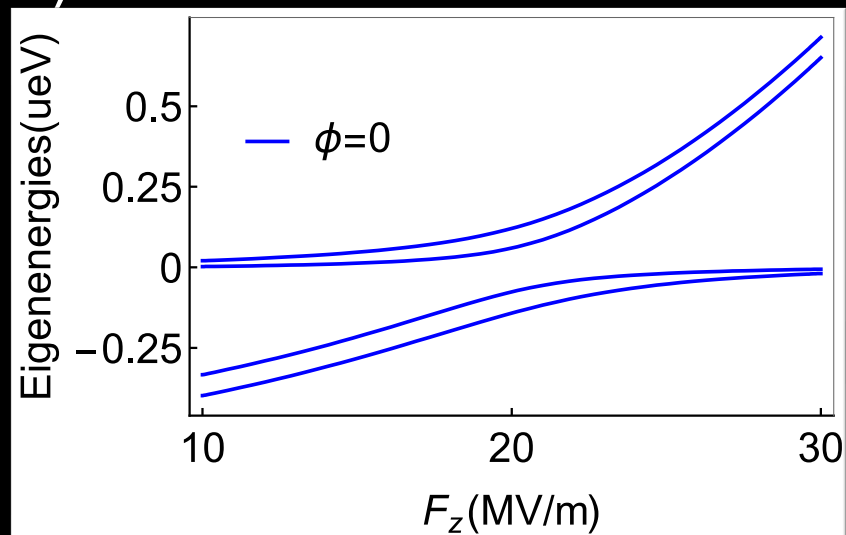
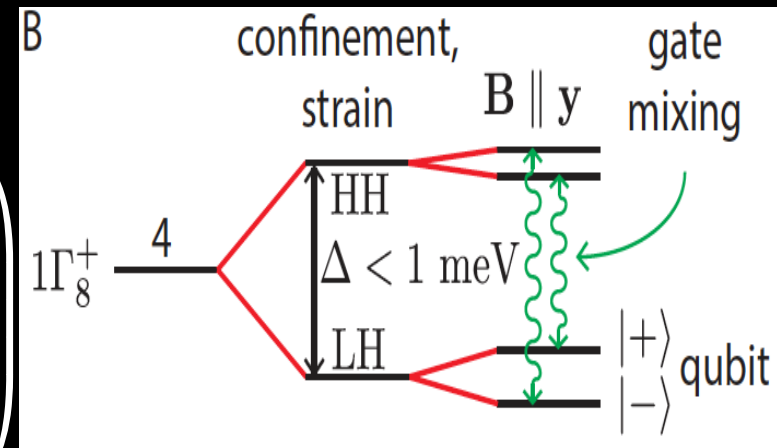




# BUILDING THE EFFECTIVE GS HAMILTONIAN: IN-PLANE MAGNETIC FIELD IN CRYSTAL AXIS

$$H_{\text{eff}} = \begin{pmatrix} \{3/2, 1/2, -1/2, -3/2\} & & & \\ 0 & \frac{\sqrt{3}}{2}\varepsilon_Z & -ipF_z & 0 \\ \frac{\sqrt{3}}{2}\varepsilon_Z & \Delta_{HL} & \varepsilon_Z & -ipF_z \\ ipF_z & \varepsilon_Z & \Delta_{HL} & \frac{\sqrt{3}}{2}\varepsilon_Z \\ 0 & ipF_z & \frac{\sqrt{3}}{2}\varepsilon_Z & 0 \end{pmatrix}$$

$$\varepsilon_Z = g_1 \mu_B B$$



# HAMILTONIAN IN THE QUBIT BASIS

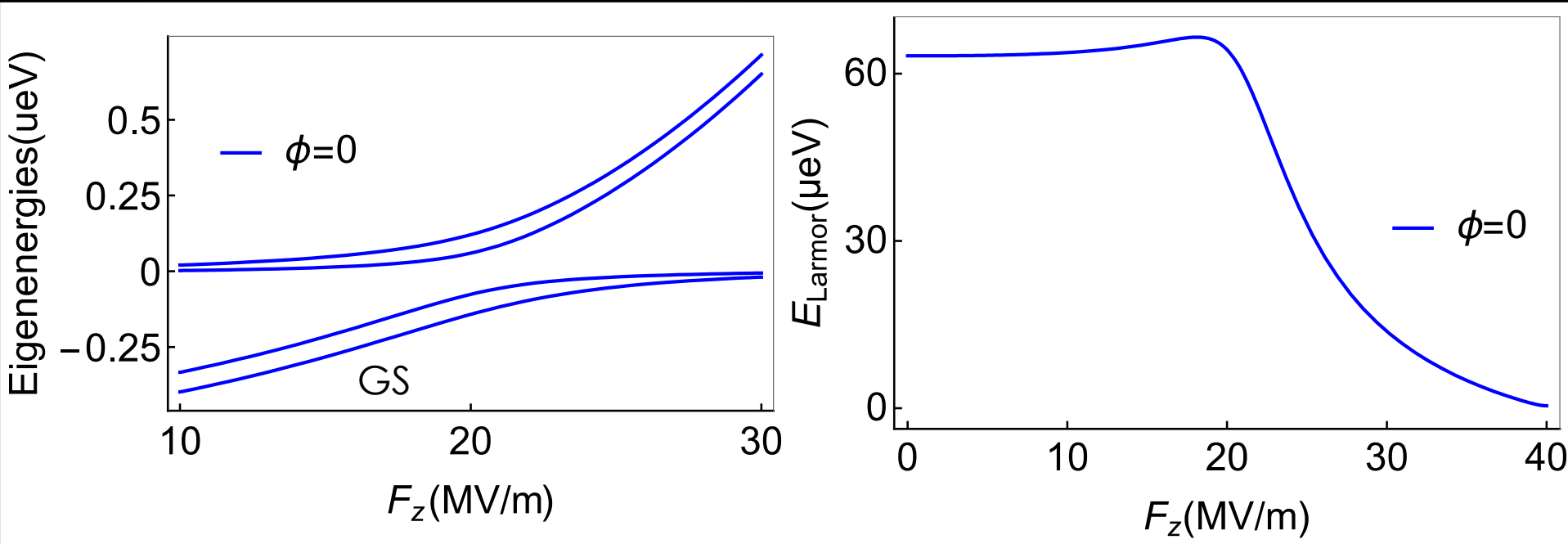
$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon Z_l & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon Z_l & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon Z_u & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon Z_u \end{pmatrix}$$

$Z_1, Z_2 \propto \varepsilon Z_0$  Qubit branch-Excited branch interaction

Splittings and mixings come from  $\varepsilon_z$ .

Depend on the LH-HH mixing and the Zeeman interaction.

# ENERGY LEVELS



Sweet spot at  $F_z^* = -\frac{\sqrt{3}\Delta_{HL}}{2p}$

Qubit is insensitive to charge noise in these sweet spots

(There is another sweet spot at  $F_z=0$ )

Salfi et al PRL (2016)

# SINGLE-QUBIT MANIPULATION

Lack of inversion symmetry: Effective Rashba interaction

$$\hat{H}_E = \begin{pmatrix} 0 & 0 & E_1 & E_2 \\ 0 & 0 & E_2 & E_1 \\ -E_1 & E_2 & 0 & 0 \\ E_2 & -E_1 & 0 & 0 \end{pmatrix}$$

Mixes qubit-excited branches

$$E_1, E_2 \propto \alpha$$

$$H_{\text{EDSR}}^{(2)} = \alpha \frac{\varepsilon_{z\alpha}}{\Delta} F_{\parallel} \sigma_x$$

This interaction is maximized at sweet spot

Single gate times: 0.2 ns at the sweet spot for acceptor at 4.6 nm.

# TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$V_{dd} = (\mathbf{v}_a \cdot \mathbf{v}_b R^2 - 3(\mathbf{v}_a \cdot \mathbf{R})(\mathbf{v}_b \cdot \mathbf{R}))/4\pi\epsilon R^5$$

$\mathbf{v}$  are spin dependent charge dipoles

$$H_{dd} \propto \alpha^a \alpha^b \epsilon_{Z_0}^a \epsilon_{Z_0}^b (\sigma_+^1 + \sigma_-^1)(\sigma_+^2 + \sigma_-^2)/R^3$$

For 20 nm :  $\sqrt{\text{SWAP}} = 2\text{ns}$

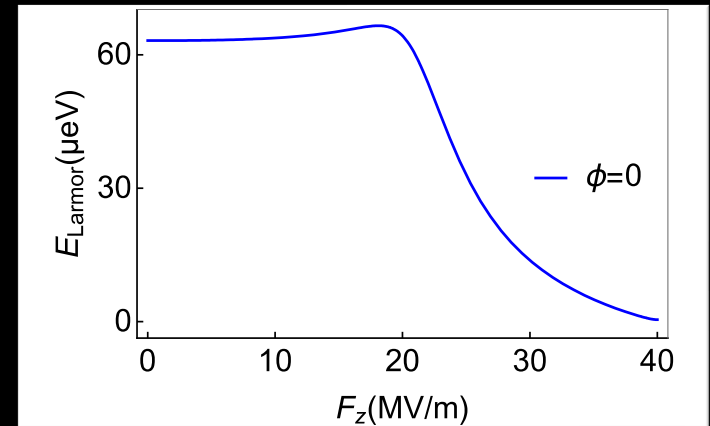
We can also use exchange for entanglement  
No valley interference effects like in donor qubits  
Circuit QED using microwave photons

# DECOHERENCE LIMITATIONS

Strong spin-orbit coupling: electrical noises cause dephasing

Qubit is insensitive to charge noise fluctuations to first order at sweet spots

$$-\frac{\sqrt{3}}{2}|\pm 1/2\rangle \mp i\frac{1}{2}|\mp 3/2\rangle$$



Phonon-induced relaxation

$$\frac{1}{T_1} = \frac{(\hbar\omega)^3}{20\hbar^4\pi\rho} C_d \left( \frac{\varepsilon Z_0}{\Delta} \right)^2$$

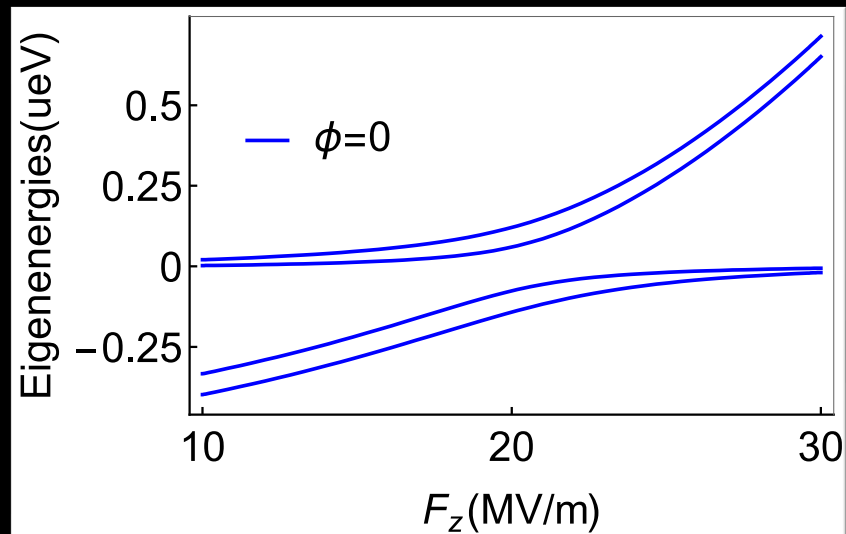
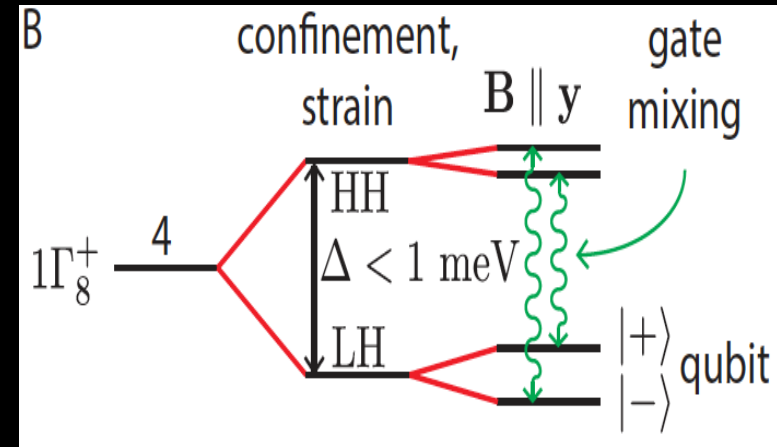
$T_1=20\mu\text{s}$  for  $B=0.5\text{T}$  at sweet spot

Still  $10^5$  operations in this time can be performed

# ARBITRARY IN-PLANE MAGNETIC FIELD

$$H_{\text{eff}} = \begin{pmatrix} \{3/2, 1/2, -1/2, -3/2\} & & & \\ 0 & \frac{\sqrt{3}}{2}\epsilon_Z & -ipF_z & 0 \\ \frac{\sqrt{3}}{2}\epsilon_Z^* & \Delta_{HL} & \epsilon_Z & -ipF_z \\ ipF_z & \epsilon_Z^* & \Delta_{HL} & \frac{\sqrt{3}}{2}\epsilon_Z \\ 0 & ipF_z & \frac{\sqrt{3}}{2}\epsilon_Z^* & 0 \end{pmatrix}$$

$$\epsilon_Z = g_1\mu_B B \rightarrow g_1\mu_B B e^{i\phi}$$

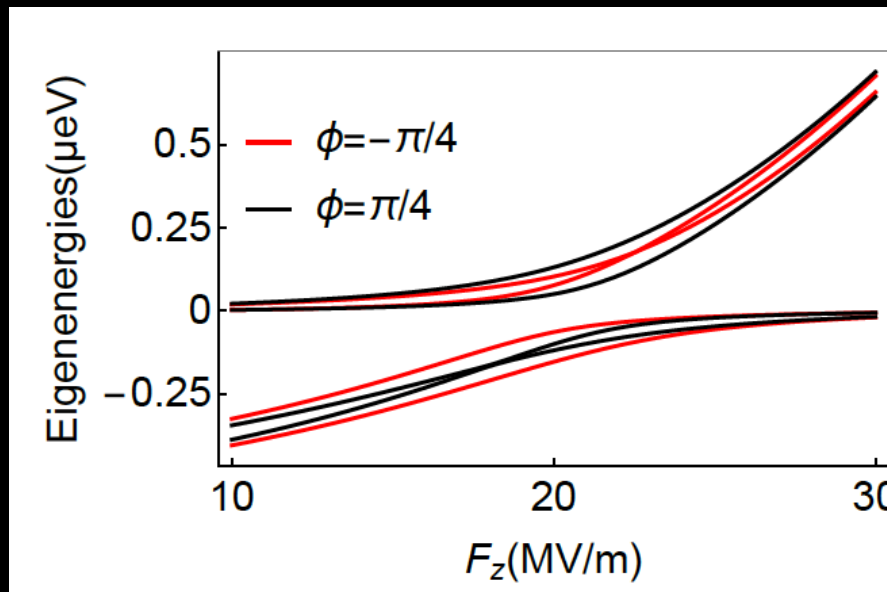


# HAMILTONIAN IN THE QUBIT BASIS

$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon Z_l & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon Z_l & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon Z_u & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon Z_u \end{pmatrix}$$

Splittings and mixings come from  $\varepsilon_z$ . They are complex functions of  $\phi$ .

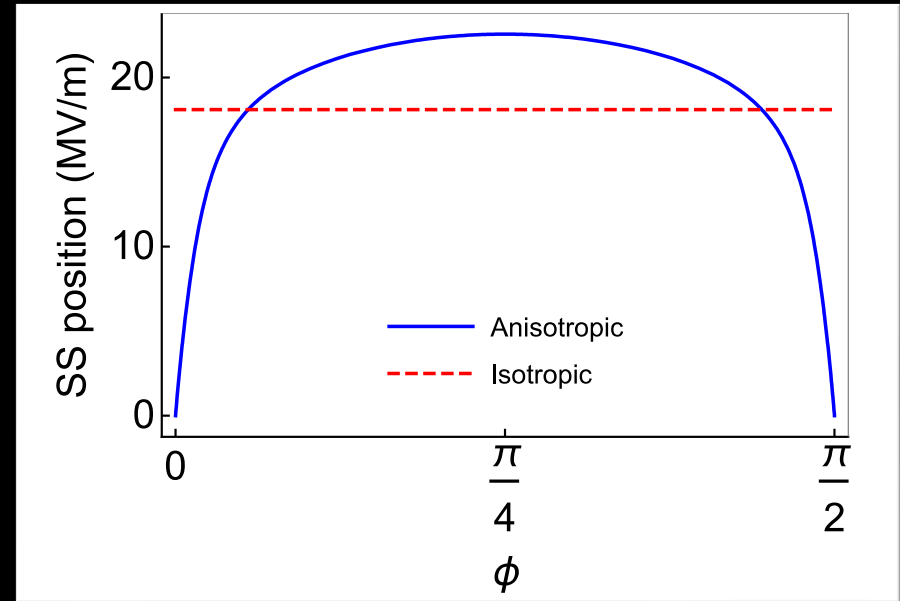
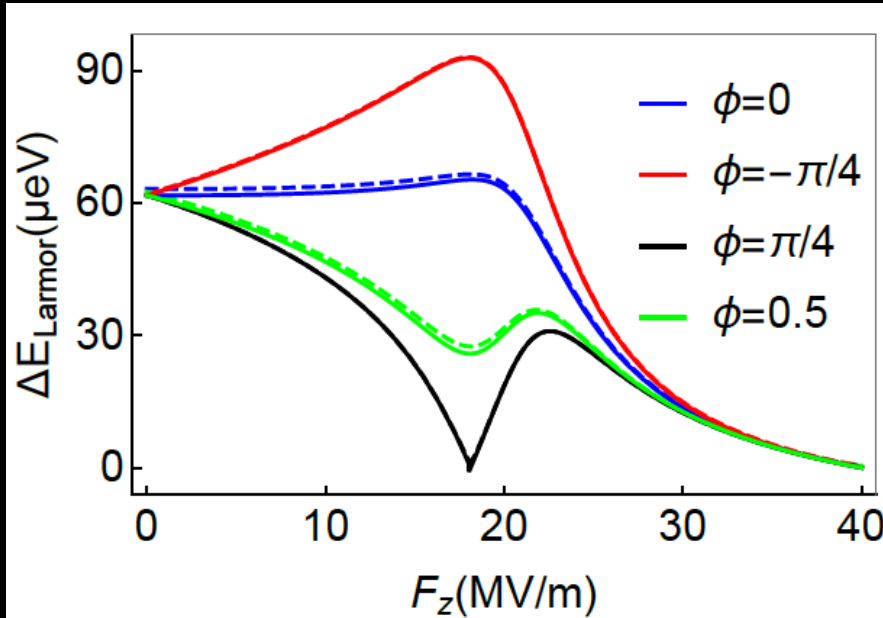
Qubit-upper states interaction terms



Strong magnetic field orientation dependence



# MAGNETIC FIELD DEPENDENCE



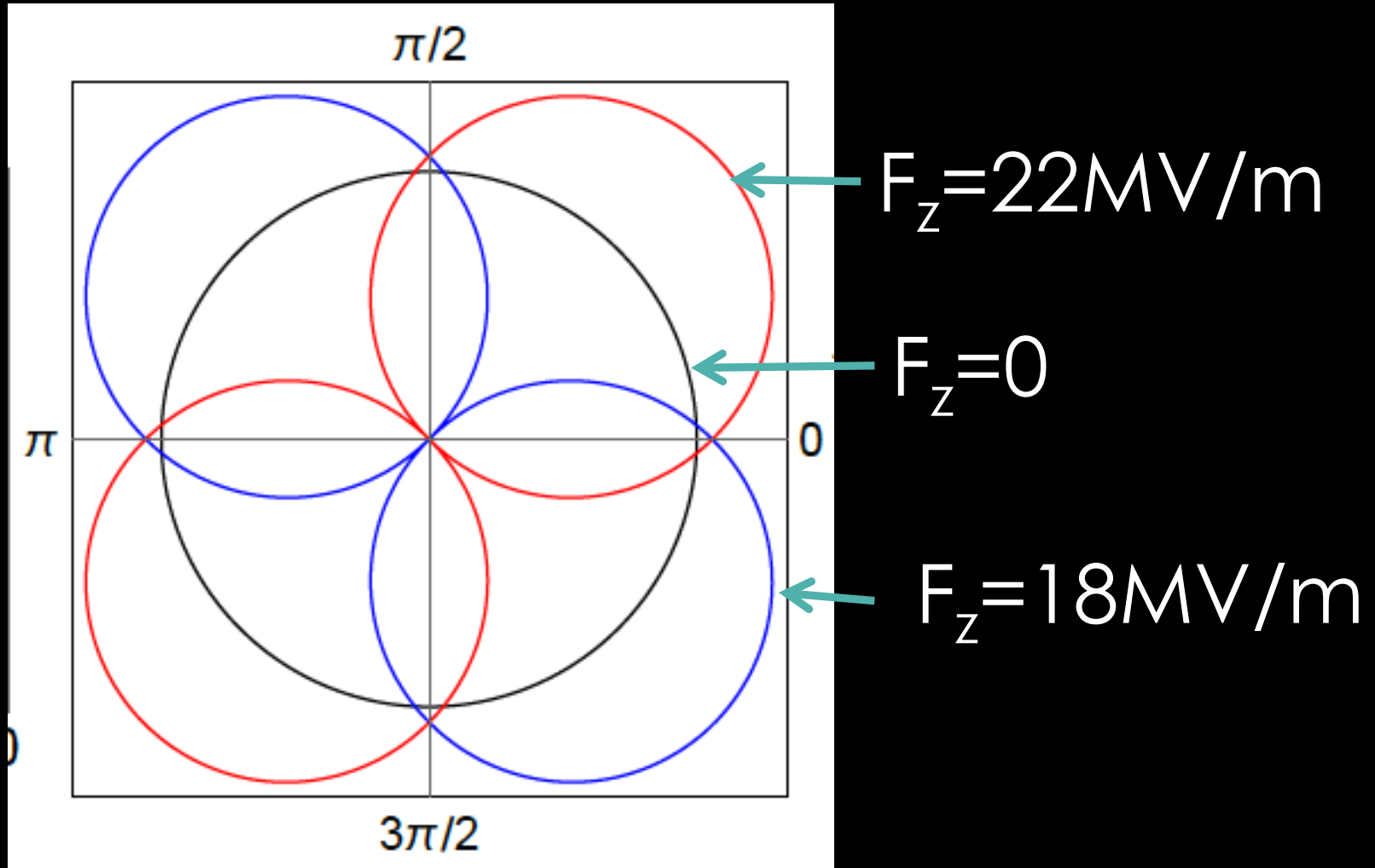
Two sweet spots:

- Isotropic sweet spot  $F_z^* = -\frac{\sqrt{3}\Delta_{HL}}{2p}$

$$-\frac{\sqrt{3}}{2}|\pm 1/2\rangle \mp i\frac{1}{2}|\mp 3/2\rangle$$

Anisotropic (dependent on  $\phi$ )  
sweet spot

# G-FACTOR IN-PLANE ANISOTROPY

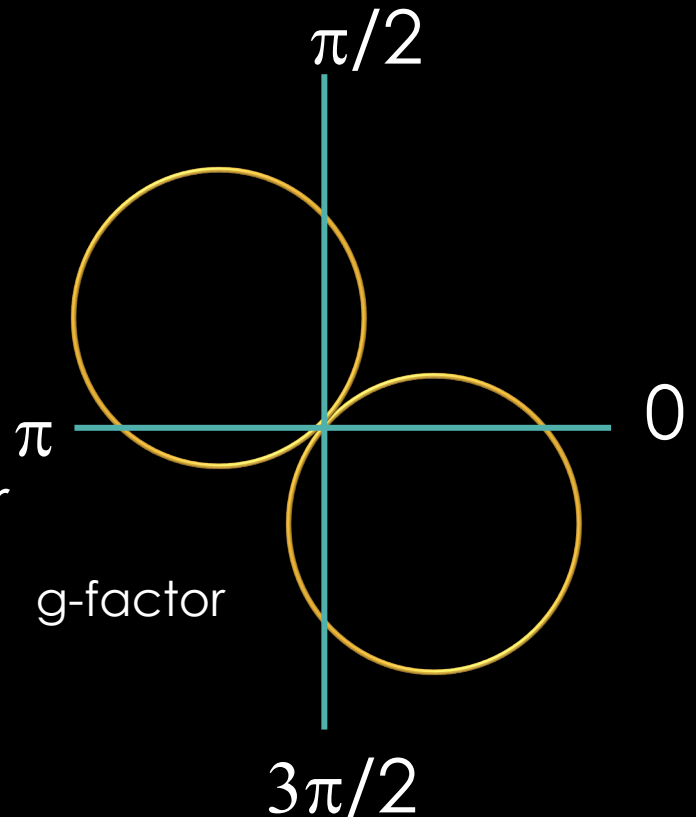


# SPIN POLARIZATION AT SWEET SPOTS

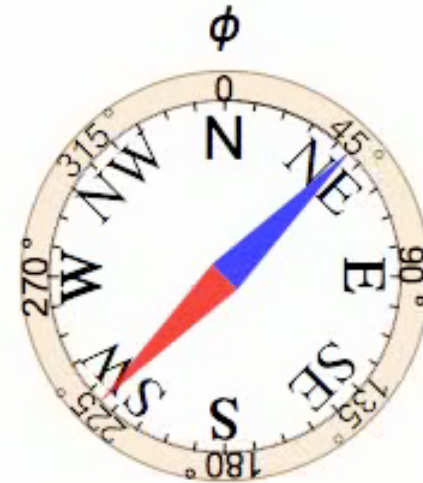
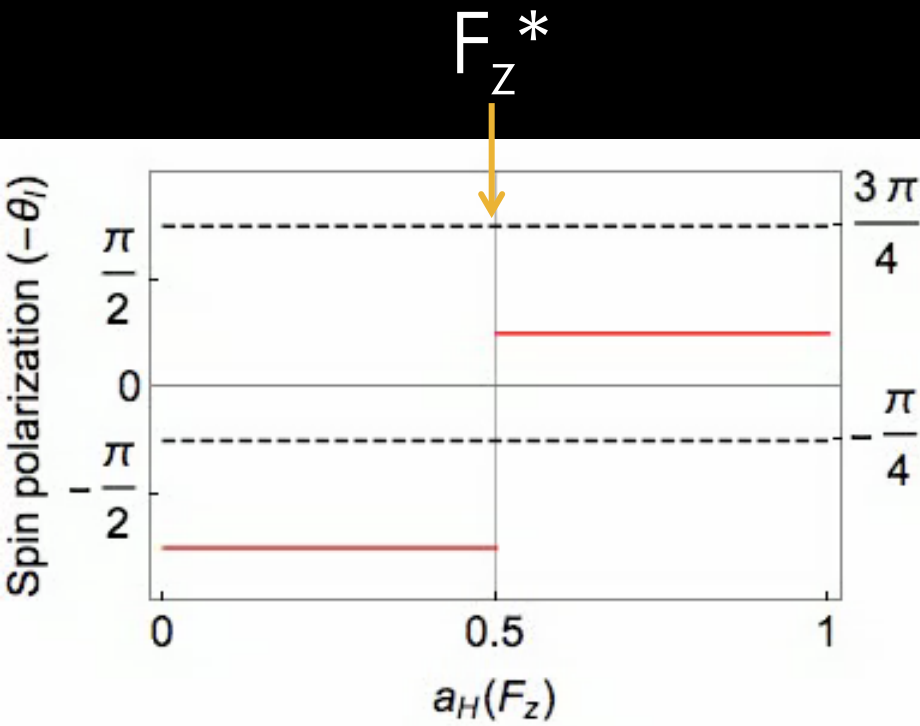
At  $F_z^*$  the spin polarization is in the  $-\pi/4+n\pi$  direction

The g-factor is 0 in the perpendicular direction and maximum in the parallel direction.

The maximum corresponds to a complete decoupling of the lower and upper branches: decoherence free subspace.



# SPIN POLARIZATION AT SWEET SPOTS



$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon Z_l & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon Z_l & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon Z_u & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon Z_u \end{pmatrix}$$

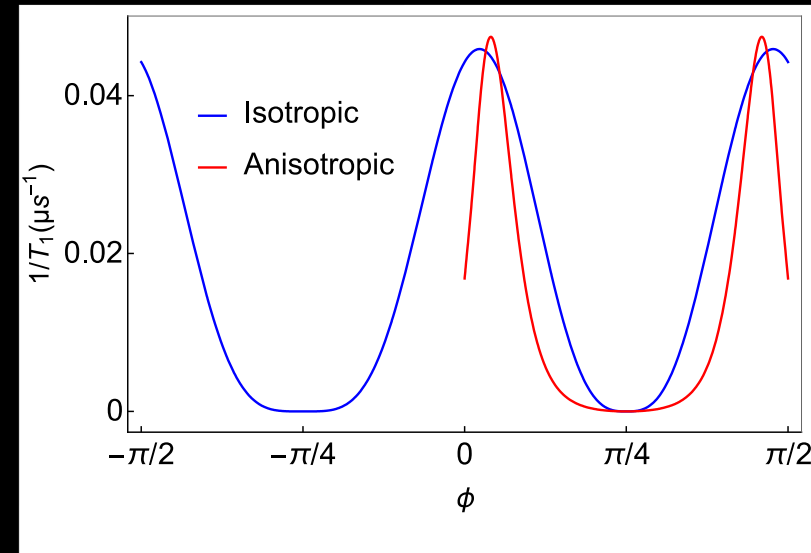
Decoherence free subspace  $[H_{\text{inter}} + H_{T_d}, H_B] = 0$

$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon Z_l & 0 & 0 & 0 \\ 0 & E_l + \frac{1}{2}\varepsilon Z_l & 0 & 0 \\ 0 & 0 & E_u - \frac{1}{2}\varepsilon Z_u & 0 \\ 0 & 0 & 0 & E_u + \frac{1}{2}\varepsilon Z_u \end{pmatrix}$$

# CONSEQUENCES: $T_1$

Phonon-induced relaxation

$$\frac{1}{T_1} = \frac{(\hbar\omega(\phi))^3}{20\hbar^4\pi\rho} C_d \left( \frac{\varepsilon_{Z_0}(\phi)}{\Delta} \right)^2$$



This mechanism is canceled to 1st order:

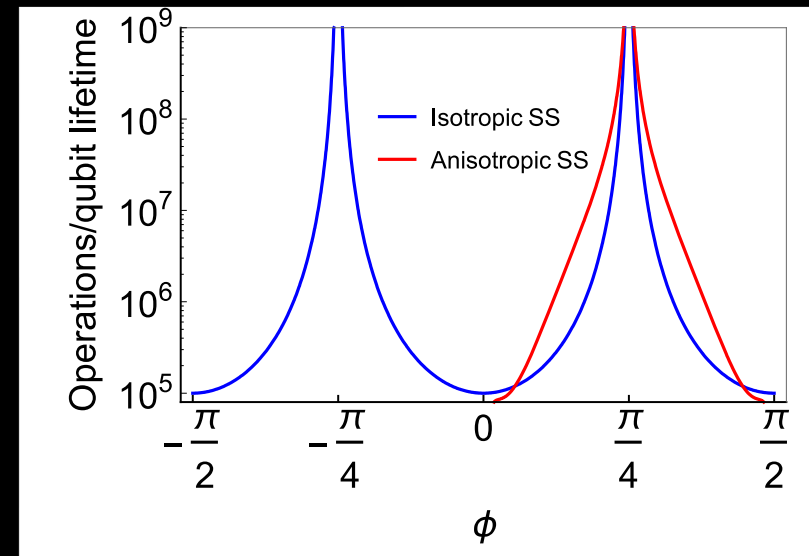
- When effective g-factor is 0
- In the DFS

# CONSEQUENCES: SINGLE-QUBIT MANIPULATION

EDSR coupling depends on  $\phi$

$$D \propto \varepsilon_{Z_0}(\phi)$$

$$\frac{1}{T_1} = \frac{(\hbar\omega(\phi))^3}{20\hbar^4\pi\rho} C_d \left( \frac{\varepsilon_{Z_0}(\phi)}{\Delta} \right)^2$$



EDSR slows down near the DFS  
T1 diverges faster than EDSR at DFS

- The number of single-qubit operations in T1 diverges at DFS
- In real devices this will be limited by second order processes.
- Still carefully choosing  $\phi$  should strongly reduce the T1 limitations

# CONSEQUENCES: TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$H_{dd} \propto \alpha^a \alpha^b \varepsilon_{Z_0}^a \varepsilon_{Z_0}^b G(F_z^a, F_z^b, \phi, \theta_E) (\sigma_+^1 + \sigma_-^1) (\sigma_+^2 + \sigma_-^2) / R^3$$

T1 diverges as fast as Hdd goes to zero at DFS: No direct improvement

But the modulating function

depends on the gate fields:  $G(F_z^a, F_z^b, \phi, \theta_E)$

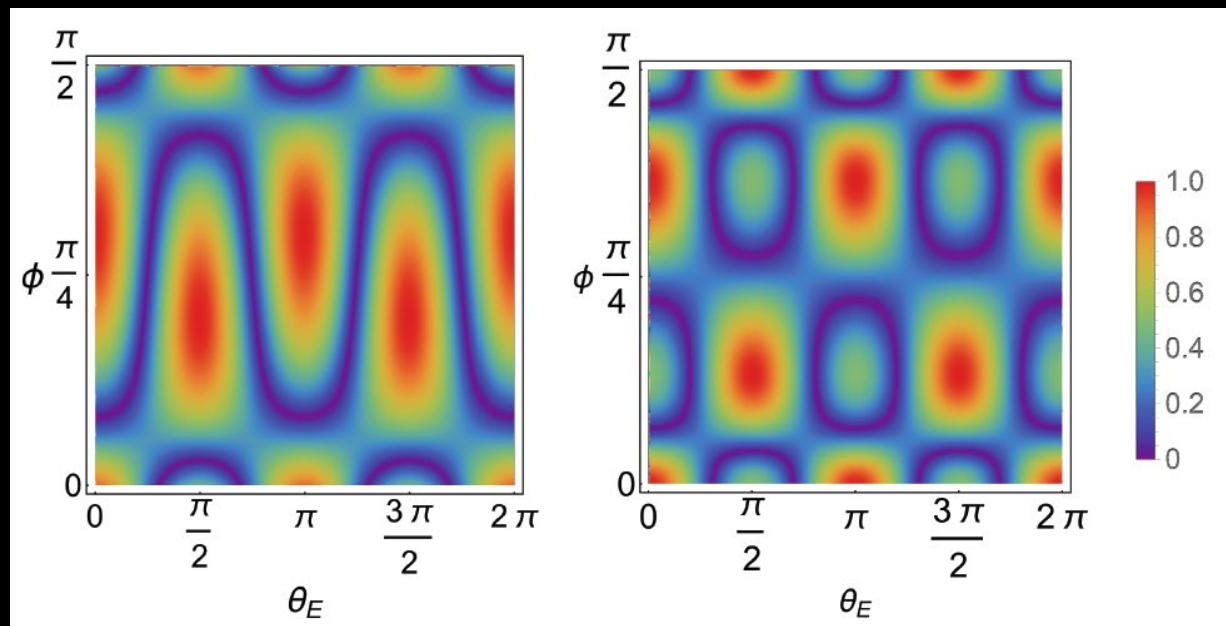
- Two qubits at isotropic sweet spot: G is maximized and isotropic
- With at least one qubit at the Anisotropic sweet spot this coupling becomes anisotropic



# CONSEQUENCES: TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$H_{dd} \propto \varepsilon_{Z_0}^a \varepsilon_{Z_0}^b G(F_z^a, F_z^b, \phi, \theta_E) (\sigma_+^1 + \sigma_-^1) (\sigma_+^2 + \sigma_-^2)$$



$$\mathbf{R} = R \cos(\theta_E) \hat{x} + R \sin(\theta_E) \hat{y}$$

# MAGIC ANGLES



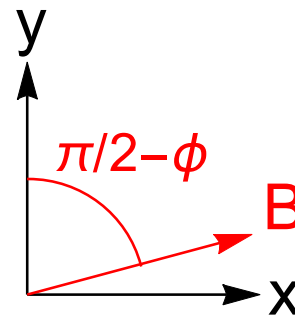
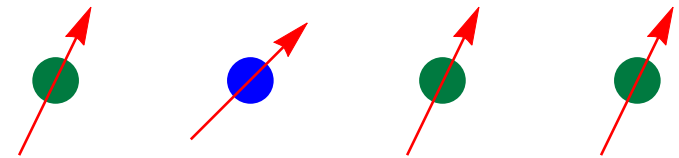
# ELECTRICAL CONTROL OF TWO-QUBIT OPERATIONS

-Two-qubit operations are tunable in one direction

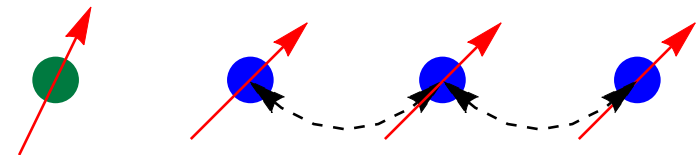
-In the other direction cQED can be used

-We devise two different protocols

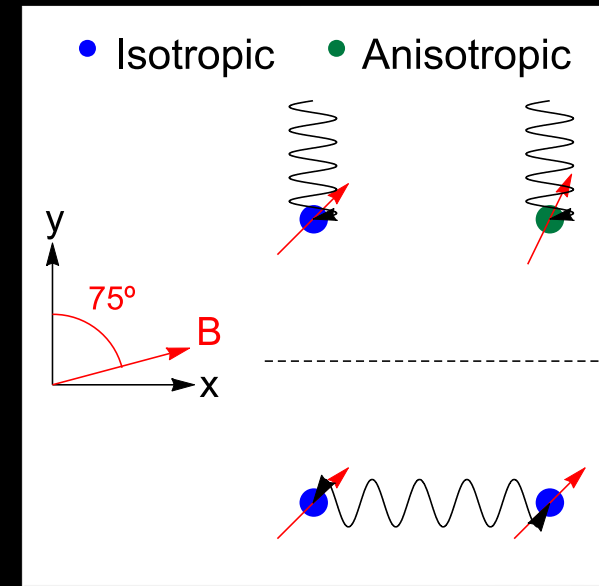
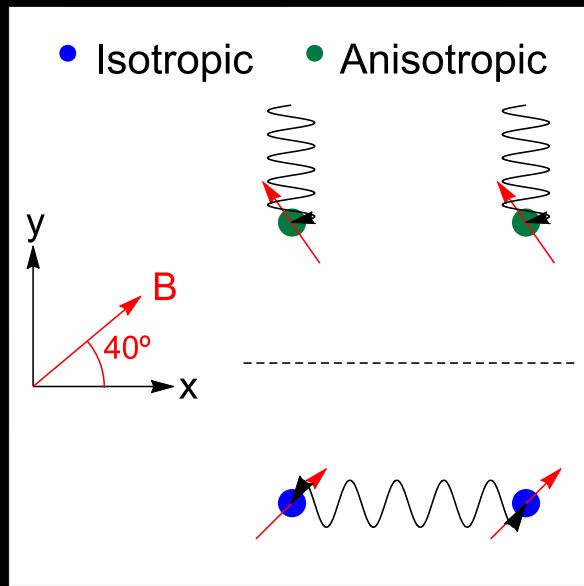
• Isotropic    • Anisotropic



Resonator



# ELECTRICAL CONTROL OF TWO-QUBIT OPERATIONS



Protocol 1:

- $\phi=40^\circ$  ( $50^\circ$ )

-Single qubit operations in ASS-ASS

-Two qubit operations in ISS-ISS

Protocol 2:

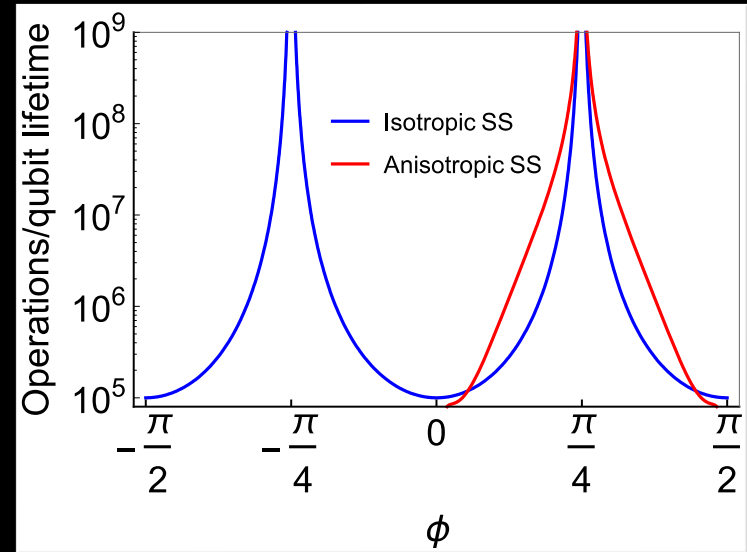
- $\phi=15^\circ$  ( $75^\circ$ )

-Single qubit operations in ISS-ASS

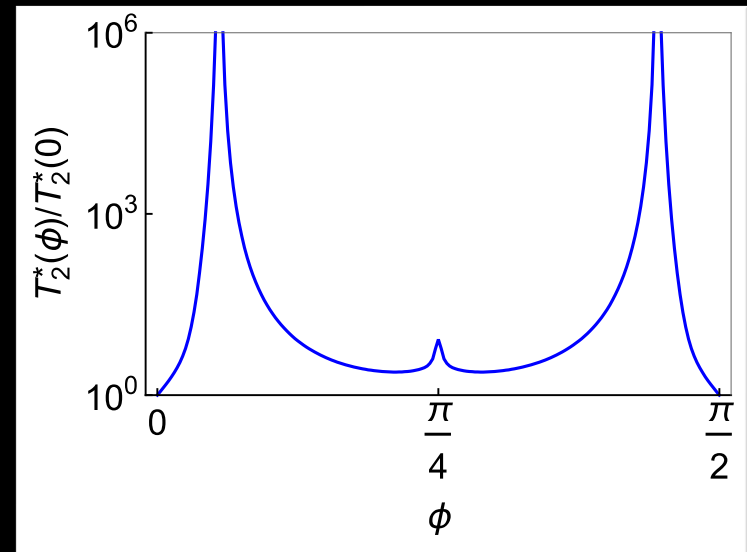
-Two qubit operations in ISS-ISS

# PROTOCOL 1 VS PROTOCOL 2

-The angles in P1 is near DFS:  
Enhanced single-qubit  
operations



-The angles in P2 implies ISS  
very close to ASS:  
Reduced charge noise  
exposure during the  
adiabatic sweep



# CONCLUSIONS

Despite the strong SOC acceptor qubits allow fast operations and yet have desirable coherence properties: Holes are coherent!

The lower local symmetry of the acceptor + spin 3/2 physics of the GS give rise to interesting magnetic phenomena:

- Dramatic in-plane g-factor anisotropy
- Decoherence Free Subspace
- Two qubit coupling anisotropy

This makes the in-plane magnetic field orientation an unexpected knob that can be used after to:

- Extend the qubit lifetime
- Modulate the two-qubit couplings by changing gate voltages